STEADY-STATE CURRENT-SHARING IN FUSES WITH ASYMMETRICAL ARRANGEMENTS

OF PARALLEL ELEMENTS

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**Summary**

The paper is concerned with calculation of the steady-state temperature distribution in fuses with multiple parallel elements, in cases where the elements have unequal radial thermal impedances. In such cases some elements run hotter than others, and their share of the total current diminishes, due to the positive temperature coefficient of resistance. The paper gives methods of calculating the radial thermal resistances and the current share between elements.

## 1. Introduction

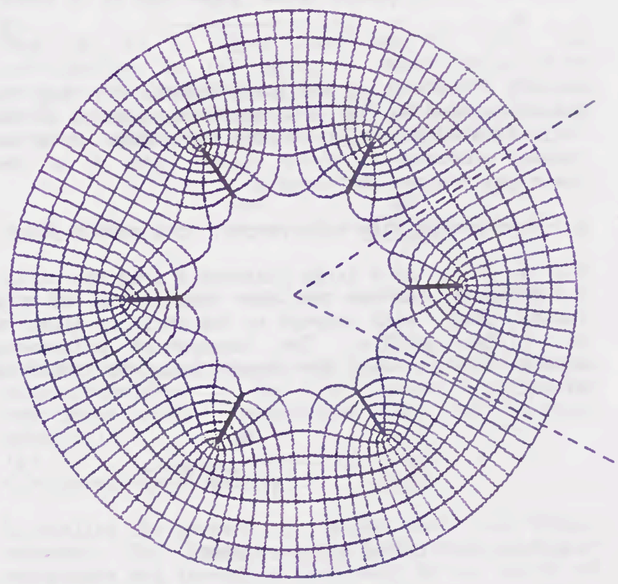
Calculation of the steady-state temperature distribution for fuses is useful for prediction of temperature rises and voltage drops under specified loading conditions, and determination of the minimum fusing current. For fuses with parallel elements, such calculations are usually based upon the assumption that the elements share the total current equally.

Most fuse designs have cylindrical symmetry, which allows reduction to a 2-dimensional (r,z) form, rather than the very complicated truly 3-dimensional form. [1,2].

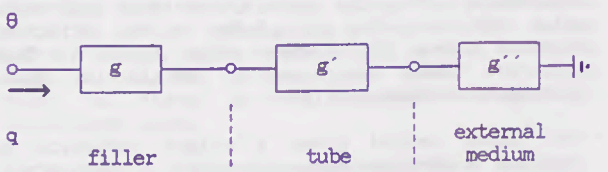
Current-carrying fuse elements lose heat firstly by axial conduction to the ends, and secondly by "radial" conduction through the filler and fuse body to the ambient. A further common assumption is to consider these two loss paths separately. This is equivalent to neglecting axial flow of heat in the filler.

This paper is principally concerned with calculation of the radial loss. Fig. 1 (a) shows the heat-flux lines and isothermals produced by six equal infinitely long strip sources of heat (computed by methods to be described later). At a relatively short distance from the strips, the isothermals become circular, and we can choose one of these to coincide with the inner diameter of the fuse tube, which permits calculation of the thermal resistance between the elements and the inside of the tube. Since the problem has cylindrical symmetry, heat flow from each element is restricted to an angle $2\pi/n$ and so the equivalent thermal resistance model shown in Fig.1(b) can be used, for a typical element. [1,2].

However, for many practical fuse designs the thermal impedance differs from element to element, e.g. when they are arranged in rows, or when one layer of elements is enclosed within another. Elements in the middle of a group get hotter than the average. Heat generation in them is increased due to the temperature coefficient, but this is offset by a decrease in current in the hotter elements, due their higher resistance. In this paper a method for calculating the radial thermal impedances for a general, asymmetric, arrangement of strips is given, together with an iterative procedure for determining the share of current between elements. Application to practical designs for fuses of finite length with notched elements is also discussed.



*(a) isothermals and heat-flux lines*




*(b) equivalent thermal resistance model, per sector*

*Fig. 1 Heated strips with axial symmetry (n=6)*

In Fig. 1 (b) the element temperature-rise is obtained from $\theta = q.(g + g' + g'')$. Calculation of $g'$ and $g''$ is straightforward [1]. The rest of this paper is concerned with the calculation of the filler thermal resistance $g$.

## 2. Temperature field due to a strip source of heat.

Fig. 2 shows an infinitely thin strip source of width $2a$ emitting a total heat $q$, located at (0,0) and lying along the x-axis. The isothermals for this case are ellipses surrounding the strip [3], which is represented as an infinitely thin ellipse.

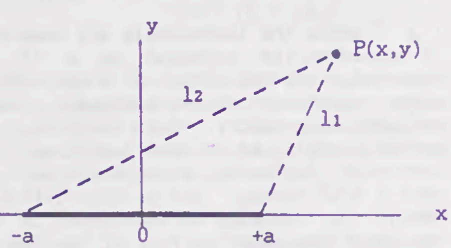



*Fig. 2 Strip source of heat located at the origin.*

The isothermals and the heat-flux lines correspond to constant values of the potential function $\theta$ and the flux function $\phi$ respectively in the inverse cosine transformation [3]

$$\theta + j\phi = \cos^{-1}(x + jy) \qquad (1)$$

From (1) the potential at P with respect to the strip is easily obtained. The form used by Sato [4] is convenient, i.e.

$$\theta = -\frac{q}{2\pi k} \ln \frac{S + \sqrt{S^2 - a^2}}{a} \qquad (2)$$

where $S = (l_1 + l_2)/2$ and the distances $l_1$ and $l_2$ are as defined by Fig. 2. S may be thought of as the "effective distance" of P from the strip. k is the thermal conductivity of the filler, and q is the power emitted per unit length.

## 2.1 Temperature rise with respect to a remote point

For a point at a large distance R from the strip, $S \approx R$ and $S \gg a$, so that the temperature of the remote point with respect to the strip is given by $\theta_R = (1/2\pi k).\ln(2R/a)$. The temperature difference between P(x,y) and the remote point is therefore given by

$$\theta_P = \frac{q}{2\pi k} \ln \frac{2R}{S + \sqrt{S^2 - a^2}} \qquad (3)$$

## 2.2 Superposition of strip sources

For a number of sources an approximation to the total temperature rise at any point P can be obtained by adding the contributions from each strip using (3), S being calculated as the "effective distance" from P to each strip in turn. This procedure has been used to obtain the field distribution shown in Fig. 1.

The above method gives a slight variation in computed temperature across the width of each strip. For a given strip the temperature rise due to its own heat generation is constant across its width, but the field due to the other strips gives rise to a variation. If the strips were perfect thermal conductors there would be no such variation. To obtain the field in this case a true Green's function would need to be used [5], with a very great increase in the complexity of the calculations.

For fuse elements, which are good but not perfect thermal conductors, and which are thin, there will be a slight variation of temperature across the strip, such as can be seen in Fig. 1, so the solution obtained by superposition method used here may well be as accurate as a true Green's function solution.

## 3. Radial thermal resistance matrix

Fig. 3 shows the isothermals and heat-flux lines for 15 heated strips arranged as a (5 x 3) array, computed using the method of superposition described above. Although the arrangement does not possess cylindrical symmetry, the isothermals again soon become circular as we move away from the strips. The outermost isothermal shown deviates by less than 2% from a true circle, and so very little error will result in choosing an isothermal in this region to represent the inner surface of the fuse tube. In the case shown the heat emitted is the same for all strips. Note that the element in the centre is hotter than the others.

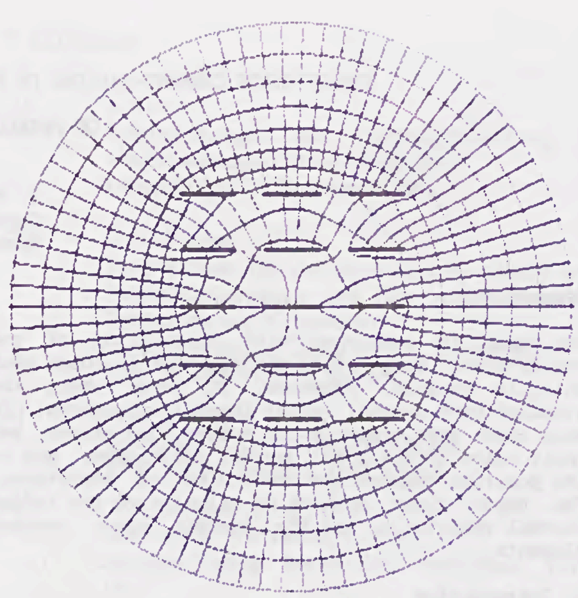


*Fig. 3 Strips without cylindrical symmetry*

Let us assume that the inner radius of the fuse tube is R. By superposition of potential contributions given by (3) we can obtain the temperature rise at any point with respect to R, but our main interest is in the temperature of the strips. These are given by :

$$\theta_1 = g_{11}q_1 + g_{12}q_2 + \dots + g_{1n}q_n$$
$$\vdots$$
$$\theta_i = g_{i1}q_1 + \dots + g_{ij}q_j \dots + g_{in}q_n$$
$$\vdots$$
$$\theta_n = g_{n1}q_1 + g_{n2}q_2 + \dots + g_{nn}q_n$$

or in matrix form,

$$[\theta] = [g]\,[q] \qquad (4)$$

where $[\theta]$ is a vector of the strip temperature rises, $[g]$ is an (n x n) thermal resistance matrix, and $[q]$ a vector of powers emitted from the strips.

The element $g_{ij}$ of the thermal resistance matrix may be interpreted as the temperature rise produced at strip i due to unit power emitted from strip j with all other powers set to zero. Using (3) this is given by :

$$g_{ij} = \frac{1}{2\pi k} \ln \frac{2R}{S_{ij} + \sqrt{S_{ij}^2 - a^2}} \qquad (5)$$

The diagonal elements of $[g]$ are all equal to $(1/2\pi k)\ln(2R/a)$, but there is some ambiguity about the off-diagonal elements, depending upon the point on strip j for which $S_{ij}$ is calculated - as already stated, use of the strip function gives rise to a variation in temperature across strip j. This can be resolved by calculating $S_{ij}$ at selected points on strip j. This is repeated for $S_{ji}$ and the geometric mean of all the distances is taken to be $S_{ij}$ (= $S_{ji}$).

$[g]$ is then a symmetric matrix, and is easily assembled given a list of the coordinates of each strip and its inclination to the x-axis.

### 4. Solution for strip temperatures

#### 4.1 Equal current sharing and zero temperature coefficient

Although this is not a realistic situation, it provides the starting point for analysis. If the strips have an electrical resistance $r_0$ per unit length and share the total current I equally the power generation is the same for all elements, i.e.,

$$q_i = (I/n)^2 r_0 = Q = \text{constant}, \quad \text{for all } i$$

From (4) the temperature rise of strip i is then

$$\theta_i = g_{i1}Q + g_{i2}Q + \ldots + g_{in}Q$$

$$\text{or} \qquad \theta_i = g_i^* Q \qquad (6)$$

$$\text{where } g_i^* = \sum_{j=1}^{n} g_{ij}$$

$g_i^*$ is the sum of row i of [g] and is the effective thermal resistance of strip i, a value which takes account of the self heating and the mutual heating. The situation can be represented by the thermal resistance model shown in Fig. 4.

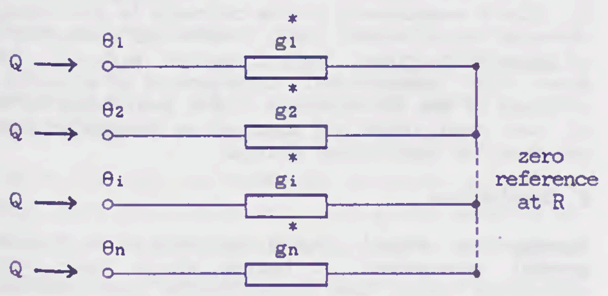



*Fig. 4 Thermal resistance model for q = constant*

The *average* temperature-rise of the n strips is

$$\theta_{av} = \frac{1}{n}\sum_{1}^{n}\theta_i = \frac{1}{n}\sum_{1}^{n} g_i^* Q$$

$$\text{i.e.} \qquad \theta_{av} = g_{av} Q \qquad (7)$$

where $g_{av}$ is the average thermal resistance per strip, and is the average of the rowsums of [g]. The average strip temperature-rise can then be found from the very simple model shown in Fig. 5.

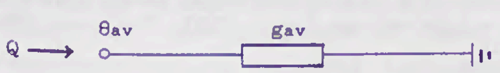



*Fig. 5 Thermal resistance model for determining $\theta_{av}$*

The individual strip temperature-rises can be evaluated if $\theta_{av}$ is known, by using (6) and (7), to give

$$\theta_i = \frac{g_i^*}{g_{av}}\,\theta_{av} \qquad (8)$$

#### 4.2 Effect of temperature coefficient alone

If we assume for the moment that equal sharing of the current is maintained, but the strip temperature coefficient ($\alpha$) is positive, the heat generation in the hotter strips will be enhanced, and the "spread" of temperatures between the strips will be increased. For strip i,

$$q_i = (I/n)^2 r_0 (1 + \alpha\theta_i) = q_0 (1 + \alpha\theta_i)$$

where $q_0$ is the "cold" strip power, $(I/n)^2 r_0$.

Thus [q] is a linear function of [θ], and substitution in (4) gives a relationship which requires the inversion of [g] to determine [θ]. A good initial approximation to [θ] can obtained however, using the "average" model of Fig. 6.


*Fig. 6 Approximate model for average temperature*

Solving the thermal network of Fig. 6 the average temperature rise is $\theta_{av} = q_0/\{(1/g_{av}) - \alpha q_0\}$. The individual strip temperatures can then be found using (8).

#### 4.3 Current-shift effect

In reality the current per strip does not remain constant. The hotter strips will have a higher resistance and therefore there will be a shift of current away from these strips to the cooler strips. This current-shift has an effect opposite to that described above due to the temperature-coefficient alone, in that it reduces the heat generation in the hotter strips, and reduces the spread in strip temperatures. The current-shift effect is about twice as strong as the effect of temperature coefficient alone.

The current in strip i is given by $r_t I/r_i$, where $r_t$ is the resistance per unit length of all the strips in parallel. This then gives

$$I_i = \frac{1}{(1 + \alpha\theta_i)\displaystyle\sum_{j=1}^{n}\frac{1}{(1 + \alpha\theta_j)}} \cdot I \qquad (9)$$

and the heat generation in each strip is then obtained as

$$q_i = n^2 q_0 \frac{1}{(1 + \alpha\theta_i)\left[\displaystyle\sum_{j=1}^{n}\frac{1}{(1 + \alpha\theta_j)}\right]^2} \qquad (10)$$

Substitution for the $q_i$ in (4) gives a non-linear set of equations for the temperatures which cannot be solved directly for [θ], but an iterative solution is possible. Substituting for $q_i$ in the i'th equation of (4), we obtain

$$\theta_i = C\left[\frac{g_{i1}}{1 + \alpha\theta_1} + \ldots + \frac{g_{in}}{1 + \alpha\theta_n}\right] \qquad (11)$$

$$\text{where } C = n^2 q_0 \Big/ \left[ \sum_{j=1}^{n} \frac{1}{(1 + \alpha\theta_j)} \right]^2$$

Equation (11) is suitable for iterative calculation of the strip temperature-rises. At the start of each iterative sweep the multiplier C is evaluated using the temperatures from the previous iteration, while the term in square brackets is evaluated using the Gauss-Seidel procedure, i.e. using the latest available estimates of $\theta_i$. The process converges rapidly.

Typical results are given in the table below, which is for an array of 15 strips positioned as shown in Fig. 3, carrying a total current of 300A. The strips were assumed to be 4mm x 0.1mm silver in a tube with R = 20mm and k = 4 x $10^{-4}$ $Wmm^{-1}degC^{-1}$.

| | | |
|---|---|---|
| 21.34 / 90.83 | 20.01 / 111.68 | 21.34 / 90.83 |
| 19.98 / 112.27 | 18.64 / 136.38 | 19.98 / 112.27 |
| 19.59 / 118.94 | 18.25 / 144.18 | 19.59 / 118.94 |
| 19.98 / 112.27 | 18.64 / 136.38 | 19.98 / 112.27 |
| 21.34 / 90.83 | 20.01 / 111.68 | 21.34 / 90.83 |

In the table the figure above the line is the strip current in A, while the lower figure is the temperature rise in degC. Equal current sharing would give 20A per strip, but the current in the centre strip is significantly lower than this. Despite its lower current the centre strip is much hotter than those at the corners.

## 5. Application to practical designs

The methods described above are for infinitely long strips, for which the heat loss is solely radial. Practical fuse elements are of finite length, and the axial heat loss along the elements to the ends is considerable. Furthermore practical fuse elements usually have reduced sections along their length, so that the element width also varies in the axial direction.

Nevertheless the ideas in section 4 can be used to obtain an approximate solution for real fuses with asymmetrical arrangements of elements, by the following procedure :

a. From the coordinates of the elements evaluate [g] and hence $g_{av}$ using the full widths of the elements in the calculation.

b. Replace the actual asymmetric arrangement of elements by an *equivalent circular arrangement* with the same number of elements, like the one shown in Fig. 1. The *only* parameter to be determined for the equivalent circular arrangement is the radius of the circle around which the elements are to be situated ($R_p$). The basis for determining $R_p$ must be that the average thermal resistance is the same as for the real fuse. $R_p$ can be found by a very simple iterative process.

c. Solve for the complete temperature distribution in the equivalent circular arrangement, by any convenient method, e.g. the finite difference or finite element method. Then by integration axially along the element in the equivalent circular arrangement, determine the average element temperature-rise, $\theta_{av}$.

d. Calculate the value of $q_0$ corresponding to this value of $\theta_{av}$. From Fig. 6 this is found to be

$$q_0\ (av) = \frac{\theta_{av}}{g_{av}\ (1 + \alpha\theta_{av})} \qquad (12)$$

e. Use the iterative procedure (11) to generate the "spread" of average temperature-rises for the real element system.

f. Assuming that the "shape" of the axial temperature distribution is the same for all elements, determine the maximum element temperature, the element resistances and the volt drop, and the current in each element.

The justification for using the full element width for evaluating [g] and $R_p$ is that the bulk of the heat loss to the filler is lost from the full strip, since the notch zones only constitute a small fraction of the total element surface area.

It has not been possible to verify these predictions by direct measurement of the currents in individual elements within a real fuse. However the predictions of temperature-rises, volt-drops and m.f.c.'s for fuses with asymmetrical arrangements of elements, obtained by the above method, have been found to be of the same order of accuracy as the predictions obtained for symmetrical designs.

## 6. Conclusions

Steady-state radial losses by conduction from an general arrangement of heated strips have been calculated using the inverse cosine transformation to represent each strip and superposition to obtain the total temperature-rise and heat flux. From this a formulation for the radial thermal resistance matrix has been developed, which relates the strip temperature rises to the emitted powers.

An iterative procedure may be used to determine the strip currents, and this can be used in modified form for real fuses with asymmetrical arrangements of elements. By replacing the real system of elements by an equivalent circular arrangement, the need for solving a truly 3-dimensional problem is avoided.

## 7. References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[5] Chorlton F. Boundary value problems in physics and engineering. (Van Nostrand 1969).