SIMULATION OF FUSELINK TEMPERATURE-RISE TESTS

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INTRODUCTION

The temperature-rise test is one of the most commonly used tests in the fuse industry. This paper is concerned with the computation of fuselink temperature rises under steady-state conditions. Heat is generated within the fuse elements by Joulean heating, and is lost axially by conduction to the endcaps and radially by conduction through the filler and fuse body. Although in the past there have been attempts to develop analytical solutions for the temperatures of the elements and hence to determine the minimum fusing current, these methods have involved many simplifying assumptions. Simulation of the temperature rise test accurately requires the following to be taken into account; heat transfer to and from the connecting cables and busbars and internal heat generation within them; heat generation within the endcaps; radiation and convection loss from the fuse body for horizontal or vertical mounting; axial heat transfer in the filler and body; the effect of non-uniform flow in the fuse elements; and M-effect processes (if used).

Some of these phenomena have been taken into account previously by using numerical methods of solution, such as finite differences [1,2]. Such methods however require vast arrays of nodes to model the 3-dimensional field accurately, and convergence is slow, requiring excessive computer time even on the largest machines available today. If the non-linear convection and radiation boundary conditions are used, convergence may be impossible.

The present paper describes a method of solution which is semi-analytical and which incorporates all of the important phenomena listed above. An analytical formula is used in the determination of the element temperature distribution, while the heat loss paths are represented by a (non-linear) lumped thermal resistance network. Using an iterative method, solutions of reasonable accuracy can be obtained very quickly.

TEMPERATURE DISTRIBUTION IN AN ELEMENT SECTION

Consider an element section of length 2ℓ as shown in Fig.1. The steady-state heat balance equation is

$$\frac{I^2\rho_o(1+\alpha T)}{S} = \frac{T}{g} - KS\frac{d^2T}{dx^2} \qquad (1)$$

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Where T = temperature rise (function of x)
x = axial position ($0 \leqslant x \leqslant 2\ell$)
I = current
ρ_o = resistivity at zero datum temperature
α = temperature coefficient
S = cross-sectional area
K = thermal conductivity
g = radial thermal resistance, per unit length.

The LHS of (1) is the Joulean heat generation per unit length, while the terms on the RHS represent the radial and axial conduction losses, respectively. It is assumed that g is constant within the section. Solving (1), subject to the boundary conditions that $T = T_L$ when $x = o$ and $T = T_R$ when $x = 2\ell$, we obtain [3]

$$T = \frac{T_L \sin a(2\ell-x) + T_R \sin ax}{\sin 2a\ell} + \frac{k}{a^2}\left[\frac{\cos a(\ell-x)}{\cos a\ell} - 1\right] \qquad (a^2 > 0)$$

$$T = T_L\left(1 - \frac{x}{2\ell}\right) + T_R\,\frac{x}{2\ell} + \frac{kx}{2}(2\ell-x) \qquad (a^2 = 0)$$

$$T = \frac{T_L \sinh a'(2\ell-x) + T_R \sinh a'x}{\sinh 2a'\ell} + \frac{k}{a'^2}\left[1 - \frac{\cosh a'(\ell-x)}{\cosh a'\ell}\right] \qquad (a^2 < 0)$$

....(2)

where $(a')^2 = -a^2$, and

$$k = \frac{I^2\rho_o}{S^2K} \quad ; \quad a^2 = \alpha k - \frac{1}{gSK} .$$

The solution for T takes three different forms, depending upon the sign of a^2. For low currents a^2 is negative and the temperature distribution is governed by the 'flat-topped' hyperbolic functions, while for high currents a^2 is positive and the distribution is related to a sine curve.

It is often necessary to calculate the heat transferred by conduction to the ends of the section. The temperature gradient at the left-hand and right-hand ends is obtained by differentiating (2), to get

$$\left.\begin{aligned} \left.\frac{dT}{dx}\right|_L &= XT_L + YT_R + Z \\ \left.\frac{dT}{dx}\right|_R &= -YT_L - XT_R - Z \end{aligned}\right\} \qquad (3)$$

where the coefficients X, Y and Z are given in the Table below.

	X	Y	Z
$a^2 > 0$	$-a \cot 2a\ell$	$a \operatorname{cosec} 2a\ell$	$\frac{k}{a} \tan a\ell$
$a^2 = 0$	$-\frac{1}{2\ell}$	$\frac{1}{2\ell}$	$k\ell$
$a^2 < 0$	$-a'\coth 2a'\ell$	$a'\operatorname{cosech} 2a'\ell$	$\frac{k}{a'} \tanh a'\ell$

These formulae refer to a single element section, the whole element being regarded as the series combination of a number of such sections (Fig.1a).

RADIAL THERMAL RESISTANCE

Referring now to Fig.1(b), we will calculate the radial thermal resistance g for a section of width w_i and thickness t_i. If there are n elements in parallel we need to compute g for a sector of angle $(2\pi/n)$. As there is no analytical formula for a strip element we may treat the element as an equivalent round wire. This method, due to Guile [4] gives the radius of the equivalent wire as

$$r_e = \frac{w_i}{2} \cdot \frac{x}{x + \sqrt{x^2 - 1}} \tag{4}$$

where $x = 2r_a/w_i$ (assuming an isothermal at r_a).

Analysis of the thermal field due to a parallel circular array of wires, using the theory of conjugate functions [5], gives the thermal resistance per sector of the filler as

$$g_f = \frac{1}{2\pi k_f} \ln \frac{r_a^n - r_p^n}{\left(r_p + r_e\right)^n - r_p^n} \tag{5}$$

where k_f is the filler thermal conductivity and r_p is the pitch circle radius. (5) is quite general, since for $r_p = 0$ and $n = 1$ it reduces to the well-known formula for a single, central wire.

The body thermal resistance is given by

$$g_b = \frac{1}{2\pi k_b} \ln \frac{r_b}{r_a} \tag{6}$$

where k_b = thermal conductivity of the body, and the external thermal resistance is

$$g_{ext} = \frac{1}{2\pi r_b h_b} \tag{7}$$

where h_b is the effective surface heat loss coefficient, which may be approximated by [6]

$$h_b = C\left(\frac{T_b}{2r_b}\right)^{0.25} + \varepsilon\sigma\left(T^4_{b\,(abs)} - T^4_{amb\,(abs)}\right) \tag{8}$$

where C is a convection constant, ϵ is the body emissivity and σ is Stefan's constant.

The total thermal resistivity is therefore

$$g = g_f + g_b + g_{ext} \tag{9}$$

The problem is that we cannot explicitly calculate g_{ext}, since the convective and radiative losses depend upon the body temperature T_b, which is unknown. Furthermore g_{ext} is a non-linear function of temperature. As will be described later, this problem can be overcome by making an initial guess for T_b, and subsequently correcting this value.

EFFECT OF FIELD DISTORTION The analysis leading to equation (2) assumes that the Joulean heat is generated uniformly throughout the section, but in fact this is not so. Fig.2 shows a sketch of the current-flow field near a typical neck-shoulder transition region of a fuse element. The distortion of the field gives rise to additional heating which would require a full two-dimensional field model to calculate accurately. However, an analytical solution exists for the additional resistance caused by this field distortion [7]. In the case shown in Fig.2, the additional resistance is given by

$$\Delta R = \frac{\rho}{2\pi t}\left[\frac{r^2+1}{r}\ln\frac{r+1}{r-1} + 2\ln\frac{r^2-1}{4r}\right] \tag{10}$$

where t is the strip thickness and r is the ratio of full section to reduced section. Fig.2 shows that almost all of the distortion occurs in the full-section, the field in the reduced section remaining almost uniform. Thus to a first approximation the effect of field distortion can be allowed for by increasing the effective resistance of the full section by ΔR, and using the analytical formulae given previously (which assumes that the additional heating is uniformly distributed within each section). Thus the use of complex and time-consuming finite-difference solutions is avoided.

HEAT GENERATED IN ENDCAPS For fuses with high current ratings it is not possible to obtain reasonable results without including the effect of the endcap resistance. This varies with construction but typically consists of three components; the resistance of the inner cap, from the element ends to the outside of the cap (obtained using a formula similar to (5)); the resistance of the outer cap (assumed to touch the inner cap only at the periphery); and then the resistance of the tag up to a chosen point where the current is assumed to leave the fuse and at which the volt drop measurements are assumed to be taken.

By this, or a similar process, the cold endcap resistance is found. During computation, when the endcap temperature rises, the resistance is corrected by use of the appropriate temperature coefficient.

HEAT LOST TO END ASSEMBLIES Heat is lost from the end assemblies by radiation and convection from the endcaps, tags, nuts, and busbars to which the fuselink is connected, and also by conduction into the connecting cables. The latter component is lost radially from the surface of the cable until at some large distance from the fuselink a constant sink temperature is reached. This sink temperature is determined only by the heating of the cable itself. In the present method, all of these processes are represented by a lumped thermal impedance G_{ec}, which is the ratio of the endcap temperature rise to the total heat input to the end assemblies. Like the external body resistance, G_{ec} has a non-linear dependence upon the endcap temperature and therefore an initial estimate is made based upon some assumed temperature value.

SOLUTION METHOD A method will now be described for linking together the various models as described previously. Firstly consider the axial temperature distribution, and the conditions at the junction between the i'th section and the (i+1)'th section (see Fig.1). The axial heat balance requires that

$$S_i \left.\frac{dT}{dx}\right|_R^i = S_{i+1} \left.\frac{dT}{dx}\right|_L^{(i+1)} \qquad (11)$$

Now using (3), we obtain

$$S_i Y_i T_{i-1} + (S_i X_i + S_{i+1} X_{i+1}) T_i + S_{i+1} Y_{i+1} T_{i+1} = -S_i Z_i - S_{i+1} Z_{i+1} \qquad (12)$$

If there are N sections, an equation of this form can be written for each junction point except the ends (i=1 and i=N+1), where the effect of the end assemblies must be considered. At the ends a heat balance as shown below must be considered.

Heat source	Heat loss
1. Input to endcap from all parallel elements by conduction.	To busbar metalwork and cables via a thermal resistance G_{ec}.
2. Internal Joulean heating in endcap.	
3. Axial conduction to endcap through filler and body.	
4. Convective input to top cap (vertically-mounted fuses only).	

Sources 3 and 4 are allowed for by the following method. A simple lumped thermal resistance network (passive) is established with resistors R_1, R_2 and R_3, where

R_1 = total thermal resistance, element-to-body

R_2 = total exterior thermal resistance, body-to ambient

R_3 = body centre-to-endcap thermal resistance.

R_1 and R_3 are constant while R_2 is a function of the body temperature. Using this network, for a given element temperature distribution, a new value of body temperature can be computed. The axial heat input to the end-cap can be then simply calculated and the convective input to the top cap for vertical fuselinks is assumed to be a fraction f of the total loss from the body, due to the upward motion of the air. Applying these conditions to the endcaps gives two further equations, which together with the equations (12) give a matrix equation

$$[Q]\,[T] \;=\; [R] \qquad (13)$$

[Q] is a known square matrix of order (N+1)

[R] is a known (N+1) vector

[T] is the unknown (N+1) vector of temperatures.

Solution of (13) gives the temperatures at the section junctions. The temperature at any location within a section can then be found using the analytical expressions (eq.2).

[Q] is a triadiagonal matrix, which permits minimal computer storage and very fast efficient solutions. The elements of [Q] and [R] are given by the following relationships:

major diagonal elements of [Q]

$$\left\{\begin{aligned} q_{11} &= S_1X_1 - \frac{1}{n.n_bKG_{ec}} - \frac{1}{KR_3} \\ &= q_{N+1,N+1} \\ q_{ii} &= S_iX_i + S_{i+1}X_{i+1} \end{aligned}\right.$$

minor diagonal elements of [Q]

$$\left\{\begin{aligned} q_{i,i+1} &= S_{i+1}Y_{i+1} \\ q_{i+1,i} &= q_{i,i+1} \end{aligned}\right.$$

$$q_{ij} = 0 \quad , \quad \text{otherwise}$$

elements of [R]

$$\left\{\begin{aligned} r_1 &= -S_1Z_1 - \frac{I^2R_{ec}}{nn_bK} - \frac{T_b}{KR_3} - \frac{fT_b}{KR_2} \\ r_i &= -S_iZ_i - S_{i+1}Z_{i+1} \\ r_{N+1} &= -S_NZ_N - \frac{I^2R_{ec}}{nn_bK} - \frac{T_b}{KR_3} \end{aligned}\right.$$

where n_b is the number of fuse bodies in parallel.

Solution is achieved by using the iterative scheme shown in Fig.3. First values of the body and endcap temperatures are assumed, which permits evaluation of all the lumped thermal resistances. Solution of the matrix equation (13) then gives the complete element temperature distribution, which includes a new value of endcap temperature. Use of the thermal resistance network then gives a new estimate of the body temperature. This procedure is repeated until the body and endcap temperatures converge. Convergence, however, is not guaranteed, owing to the basic non-linearity of the problem, and in fact for high applied currents convergence becomes quite difficult with certain fuse designs unless under-relaxation is used. In this method, the new value of, for example, body temperature is calculated as

$$T_b' = T_b^{(old)} + u\left(T_b^{(new)} - T_b^{(old)}\right) \qquad (14)$$

where $u<1$, is the underrelaxation factor. Another factor which critically affects convergence is the value chosen for the initial estimate of body temperature. Fig.4 illustrates the effect of these parameters upon the number of iterations to converge for a particular fuse.

Use of an initial body temperature rise of 500°C and an underrelaxation factor of 0.2 has been found to give satisfactory convergence to date for all fuses studied, from zero current up to the minimum fusing current.

M-EFFECT After the element temperature distribution has been calculated the maximum temperature is compared with the melting temperature. For fuses with M-effect the situation is more complicated. It is not sufficient to consider that element rupture will occur if the temperature exceeds the melting point of the M-spot material. Even if this temperature is exceeded, the rate of diffusion may be so slow that rupture does not occur within the conventional time (t_c).

To account for this, a simple model for the diffusion process has been used, in which the depth of penetration of the diffusion interface is assumed to be [8]

$$z = \left[D_o t \exp(-Q/T)\right]^{\frac{1}{2}} \qquad (15)$$

where Q is the activation energy constant and D_o is a diffusion constant. This permits the calculation of an 'effective melting temperature' which is the temperature which must be sustained in order for the element to rupture in the conventional time. If the element thickness at the M-spot location is z_c, the effective melting temperature is then obtained from (15), as

$$T_m = \frac{Q}{\ln\left(\frac{D_o t_c}{z_c^2}\right)} \qquad (16)$$

T_m is given from (16) in Kelvin. Values of D_o and Q have been determined for several commonly-used types of M-spot material. This simple model appears to represent accurately the important processes affecting M-spot operation, and to date has given good results.

PROGRAM 'TRISE' AND TYPICAL RESULTS The models described have been incorporated in an interactive computer program TRISE which is one of a suite of programs [9] for fuse simulation. The program computes the temperature distribution for a fuse selected by the user, when tested under standard or non-standard conditions. A simulated test report is then output giving the predicted values of cold resistance, power loss, mV drop, end-cap temperature, body temperature, hotspot temperature plus a series of comments.

The Table below gives a comparison of predicted values at rated current with test values, for fuses selected from a low-voltage industrial range. In each case the first figure is a measured value and the second is the computed value.

Fuse rating (A)	Cold resistance (μΩ)	mV drop	endcap rise (^oC)	body rise (^oC)
32	2950	110.7	43.0	39.0
	2570	110.7	42.3	44.5
100	610	70.0	32.0	29.0
	561	69.7	37.0	38.4
200	284	77.3	44.7	47.5
	299	77.8	46.9	50.4
315	218	91.6	58.6	64.0
	205	86.8	53.2	60.7
630	90	77.0	62.5	56.5
	90	71.1	56.6	56.7

The results above are typical in that the cold resistance and mV drop values are much more accurate than the temperature-rise values. This is not surprising as the latter values are very sensitive to slight changes in the test environment, such as the presence of draughts. The element temperature distributions are much more accurate and result in predicted minimum fusing currents very close to those which may be estimated from the measured time-current characteristic.

CONCLUSIONS

The paper has described a method for simulating a temperature-rise test on a fuselink. The method uses analytical solutions as far as possible together with a linking iterative algorithm. The method has been found to converge for all fuses tried so far. Initial results correspond well with those observed in the test laboratory, but the program remains to be accurately 'calibrated' for all types of fuse by a more precise determination of the convection constants appropriate to the particular fuse geometry.

The structure of the program TRISE is such that improved models of the various processes involved may be substituted for the models described here, so that the accuracy of the predictions may be improved in the future.

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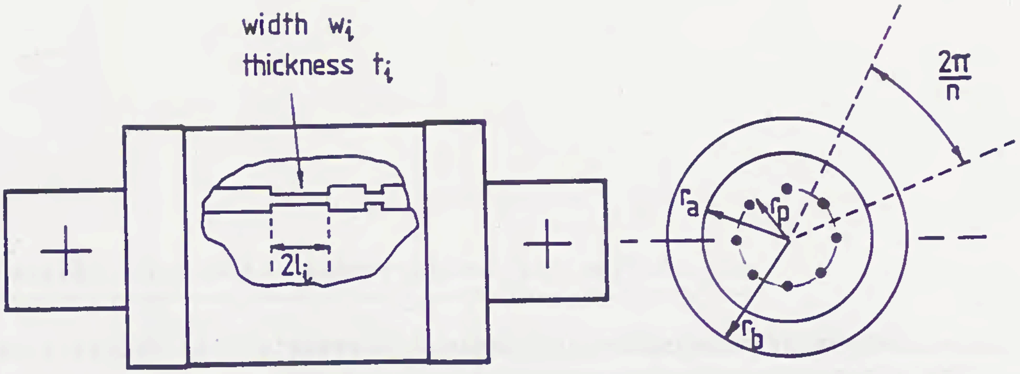

Fig 1. Basic geometry

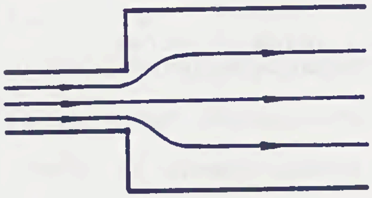

Fig 2. Current-flow field

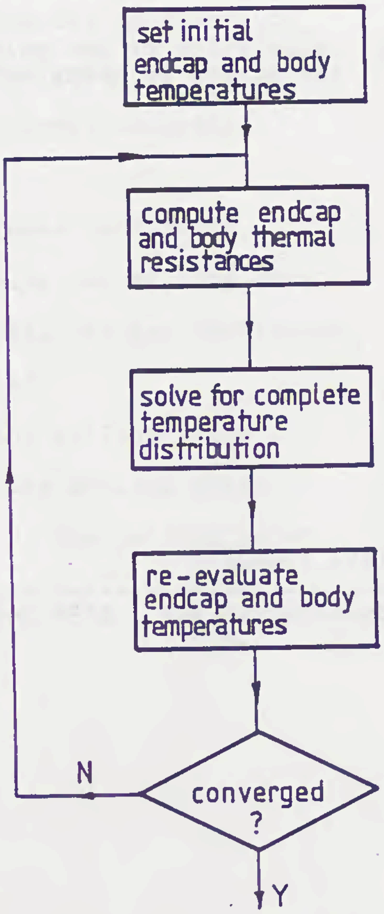

Fig 3. Iterative method

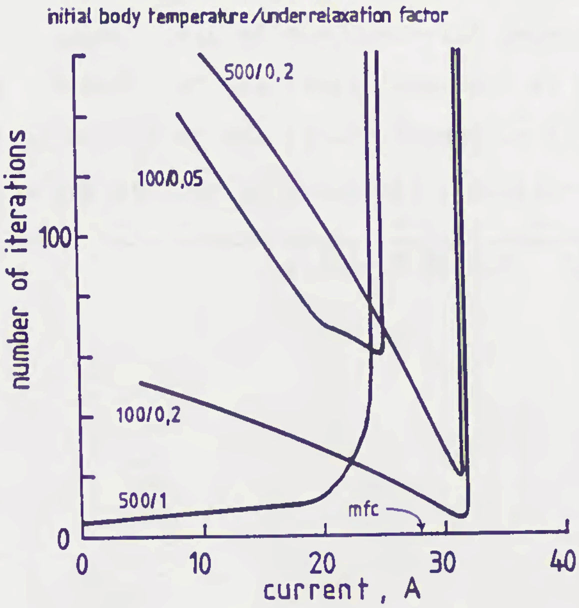

Fig 4. Convergence range

Calculations of models for adiabatic processes

1. calculation of the maximum duration of adiabatic melting

2. calculation of the melting integral value of wires and strips by means of material constants only

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Material constants: abbreviations and definitions

Ao = resistivity of the conductor at 20 degrees centigrade

A1 = first order temperature coefficient of resistivity

A2 = second order temperature coefficient of resistivity

A3 = thermal conductivity k at 20 degrees centigrade

A4 = first order temperature coeff. of thermal conductivity

A5 = thermal capacity c at 20 degrees centigrade

A6 = first order temperature coeff. of thermal capacity

A7 = coeff. of linear thermal expansion

A8 = density d of the material at 20 degrees centigrade

A9 = resistivity of the liquid conductor at the melting point

Bo = melting temperature Ts of the material,degrees centigrade

B1 = latent heat of the conductor material

B3 = density of the solid conductor at the melting point

B4 = density of the liquid conductor at the melting point

W9 = resistivity of the solid conductor at the melting point

Material constants: numerical values

	dimension	Ag	Cu	sand
Ac	Ω*mm^2/mm	.000016	.00001	
A1	1/K	.00377	.0039	
A2	1/K^2	5.7 E-7	6.7 E-7	
A3	W*s/(mm*s*K)	.428	.385	.0003
A4	1/K	- .00018	- .0008	
A5	W*s/(g*K)	.237	.377	.9
A6	1/K	.00013	.00024	
A7	1/K	.000024	.000020	
A8	g/mm^3	.0105	.00895	.002
A9	Ω *mm^2/mm	.00016	.00022	
Bo	K	960	1083	
B1	W*s/g	105	214	
B3	g/mm^3	.00976	.00838	
B4	g/mm^3	.0091	.0079	
W9	*mm^2/mm	.0000801	.000103	
To	K	20	20	

1. Calculation of the maximum duration of adiabatic melting

The loss of heat by conduction is connected with the temperature equilization by a simple model. As a result of this method maximum melting times for adiabatic melting processes are caculated. The time/current characteristic of wires and strips for a defined range of melting time can be given quickly and accurately.

Ermitteln der maximalen Zeiten für adiabatisches Schmelzen.

Mit einem einfachen Modell werden Wärmeleitung und Temperaturausgleich verbunden. Aus dieser Beziehung werden Schmelzvorgänge und Schmelzzeit für adiabatische Vorgänge bestimmt. Damit können Kennlinien für diesen Bereich sehr genau und schnell ermittelt werden.

Loss of heat and temperature equalization

dt=time interval
dT=deviation of temperature
T=temperature difference
x=length of wire section
d=density of the material
k=thermal conductivity
c=thermal capacity
m=mass of wire section
q=cross section of wire

We use the classical theory of the conduction of heat.
We consider only the conduction of heat along a wire. Heat losses from the wire to the environment, i.e. quartz sand, by conduction or radiation are neglected because they are remarkably smaller.
On a homogenous long wire there is a section 1, equal in length and adjacent to a section 2. The section 1 shall be warmer than the section 2. The temperature difference shall be T degrees.
In a time dt the heat dQ1 is flowing from the section 1 to the section 2. The temperature difference T shall not alter much.
Then we find for the conducted heat

1.1 $$dQ1 = (k*q/x)*T*dt$$

A deviation dT of the temperature of intercept 1 requires an amount of heat dQ2.

1.2 $$dQ2 = m*c*dT = d*q*x*c*dT$$

We assume these amounts of heat dQ1 and dQ2 have the same value.

1.3 $dQ1 = dQ2 = dQ$

The deviation dT decreases the temperature difference T. Section 1 will be cooled. Section 2 conducts the heat dQ2 to the next section. The temperature of section 2 will therefore remain constant. We obtain an expression for the deviation of the temperature of section 1.

1.4 $dT = -dQ/(m*c) = -k*T*dt/(d*x^2*c)$

We separate the variables T and t and obtain the differential equation

1.5 $dT/T = -k*dt/(d*x^2*c)$

Integration of Eq. 1.5 gives the temperature difference T as a function of the time t:

1.6 $\ln(T/To) = -k*t/(d*x^2*c)$

1.7 $T/To = \exp(-k*t/(d*x^2*c))$

A small table will be more informative.
The length of the two sections is x=0.1 mm each. When the time t has passed the temperature difference has decreased from the value To to the value T.

table 1

x=0.1mm	T/To(Ag)	T/To(Cu)
t=0.1ms	0.18	0.32
t=0.01ms	0.84	0.89

Periods less than .01 ms do not alter very much the temperature difference T for lengths of x=.1 mm.

In the example mentioned above we have calculated with material constants of the temperature value 20 degrees centigrade. This limitation does not influence the result very much.

With this simple model maximum melting times for adiabatic processes in wires can be predicted very quickly.

In the following a more handy formula will be given. A wire of the length 2*x is heated up by a current. The temperature of the wire will be the highest in the middle and the lowest at both ends. We cut the wire in two sections with the length x.

In order to have maximum time values tm the length of section 2 is set to zero.We solve the formula 1.6 and obtain the maximum time tm

1.8 $$tm = (c*d/k) * x^2 * \ln(To/T)$$

(Ag) $$tm = 5.8 * x^2 * \ln(To/T)$$

(Cu) $$tm = 8.8 * x^2 * \ln(To/T)$$

According to Eq. 1.5 the temperature deviation decreases with decreasing temperature difference T.We also find that it is necessary to have a small temperature deviation dT in order to obtain a real value tm of melting time.We accept To/T=2 for practical use.That yields the value ln(To/T)=0.7 .

$$tm = (c*d/k) * (a/2)^2 * \ln(To/T)$$

1.9 $$tm = 90 * (c*d/k) * a^2$$

(Ag) $$tm = 0.5 * a^2$$

(Cu) $$tm = 0.8 * a^2$$

The values mentioned are of the following dimensions:

c (W*s/g*K) ; d (g/mm^3) ; k (W*s/mm*s*K)

a (mm) ; tm (ms)

Thus we find

* Eq. 1.9 yields the relation between the length a=2*x *
* of a wire and the maximum time tm for a heating process *
* by current that can be handled as adiabatic process. *

table 2

	a (mm)	1	2	5	10	20	50
Ag	tm (ms)	.50	2.0	12	50	200	1200
Cu	tm (ms)	.80	3.2	20	80	300	2000

Eq. 1.9 is valid for wires,small strips or other material too.

* If we have a melting time ts less than the maximum time tm *
* we can assume the heating by current an adiabatic process. *
* In this case the melting integral will be a constant value. *

1.10 $$\int_{to}^{ts} i^2(t)*dt = constant$$

Hence we can define a virtual melting time tv as a function of a sine-wave current i(t)

1.11 $$tv = constant/i^2(t)$$

When the wire is embedded into crystallized sand we cannot expect the heat losses to be zero for great values of melting time.

We assume the heat transfer from the metal surface of the wire to the sand crystal to be faster than from one sand crystal to the next.Hence the thermal conductivity in the sand will not be influenced by the heat transfer from the metal to the sand.

According to Eq. 1.9 we calculate the maximum time tx for neglectable heat losses from the wire into the sand.A realistic value for the length a in the sand is the mean diameter of the sand crystals, e.g. a=0.4 mm.We obtain

1.11 $$tx = 86 \text{ ms}$$

Hence we may say:

* If a wire embedded in crystallized sand is heated up by a *
* current pulse i(t) and if the melting time calculated by *
* means of the melting integral value is less than 80 ms we *
* can regard the melting time value as a correct value. *

If we have a small strip embedded in crystallized sand the greater surface will allow a greater heat transfer to the sand. The simplest approximation for the calculation of the maximum melting time tm(st) of a small strip of the width b embedded in sand of crystals of the mean diameter a is obtained by

1.12 $$tm(st) = tm * a/b$$

2. Calculation of the melting integral value of wires or strips by means of material constants only.

Due to a defined current in a metallic conductor the temperature rise is derived from a simple model of an adiabatic process.A clever modification of the differential equation for temperature rise creates an equation in which a pure temperature function and a pure time function are connected with an expression of material constants of the conductor only.The combination of the temperature function and the expression of the material constants delivers very good information about the temperature rise of the conductor caused by short current pulses.As a limiting value we find the melting integral value for adiabatic melting.

Berechnen des Schmelzintegralwertes aus Materialkonstanten des Schmelzleiters.

Mit einem einfachen Modell wird die Temperaturerhöhung durch einen Strom in einem Leiter für adiabatische Vorgänge angegeben. Durch sinnvolles Umformen der Differentialgleichung ensteht eine Gleichung,in der eine reine Temperaturfunktion und eine reine Zeitfunktion durch einen Ausdruck mit Materialdaten des Schmelzleiters verbunden sind.
Durch Verbinden der Temperaturfunktion mit dem Ausdruck für die Materialdaten ergibt sich eine gute Aussage über die Temperaturerhöhung eines Leiters durch kurze Stromimpulse und als Grenzwert der Schmelzintegralwert für adiabatische Vorgänge.

2.1 Physical background

Adiabatic heating processes are calculated under consideration of the influence of the temperature to material values.

A wire has a constant mass m,a length x(T).a cross section q(T) and the resistance R(T).The wire is heated up by a current depending on the time.In a time dt an amount of heat Q is created in the wire.

2.1.1 $Q = R(T)*i^2(t)*dt$

This amount of heat Q creates a deviation dT of the temperature of the wire.

2.1.2 $Q = c*m*dT = c*d*x*q*dT$

The combination of Eq. 2.1.1 and Eq. 2.1.2 leads to

2.1.3 $R(T)*i^2(t)*dt = c*m*dT$

Hence we obtain the deviation dT caused by the heat of a current pulse.

2.1.4 $dT = R(T)*i^2(t)*dt/(c(T)*m)$

2.2 Influence of the temperature to material constants.

The mass m of the wire does not depend on the temperature.When we leave the length x and the cross section q constant then the resistivity Ro of the wire is a function of the temperature T.

2.2.1 $Ro = Ao * f1(T)$

The temperature dependent resistance R(T) can be written as

2.2.2 $R(T) = Ro * (x/q) = Ao * f1(T) * (x/q)$

The function of the temperature of the resistivity is expressed by the beginning of a power series.

2.2.3 $f1(T) = 1 + A1*(T-To) + A2*(T-To)^2$

f1(T) is replaced sufficiently correct by the approximation

2.2.4 $f1(T) = 1-A1*To + (A1-2*A2*To)*T + A2*T^2$

If a wire is heated up by a current the length and the cross section will not remain constant.Both depend on the temperature. The effect of the temperature is given sufficiently correct by the beginning of a power series too.

2.2.5 $x(T) = xo * (1 + A7*(T-To))$

2.2.6 $q(T) = qo * (1 + A7*(T-To))^2$

Eq. 2.2.5 and Eq. 2.2.6 inserted in Eq. 2.2.2 lead to a formula for the temperature dependent resistance R(T) of the wire.

2.2.7 $R(T) = Ao*f1(T)*xo / (qo * (1 + A7*(T-To)))$

The thermal capacity c depends on the temperature T too.

2.2.8 $c(T) = A5*(1+A6*(T-To)) = A5 * (1-A6*To + A6*T)$

Eq. 2.2.7 and Eq. 2.2.8 inserted in Eq. 2.1.4 lead to a formula for the temperature deviation dT by the heat Q.

$$dT = \frac{Ao*xo*i^2(t)*dt*f1(T)}{qo*(1+A7*(T-To))*A5*(1+A6*(T-To))*m} \tag{2.2.9}$$

We collect the various temperature dependent expressions.

$$\begin{aligned} f2(T) &= (1+A7*(T-To))*(1+A6*(T-To)) \\ &= 1+A7*(T-To)+A6*(T-To)+A7*A6*(T-To)^2 \end{aligned} \tag{2.2.10}$$

A short estimation with the temperature value T considered as the melting temperature Ts shows that we can neglect the last member of the sum in Eq. 2.2.10 .

$$A7*A6*(T-To)^2 = 0.00002*0.0003*1000000 = 0.006$$

The function f2(T) can be written simpler.

$$\begin{aligned} f2(T) &= 1 + A7*(T-To) + A6*(T-To) \\ &= 1-(A7+A6)*To + (A7+A6)*T \end{aligned} \tag{2.2.11}$$

Eq. 2.2.4 and Eq. 2.2.10 give the temperature function f3(T).

$$\begin{aligned} f3(T) &= f1(T)/f2(T) \\ &= \frac{1-A1*To+(A1-2*A2*To)*T+A2*T^2}{1-(A7+A6)*To+(A7+A6)*T} \end{aligned} \tag{2.2.12}$$

We will have the expression f3(T) compact by the abbreviations

$$\begin{aligned} W1 &= 1-A1*To & W3 &= A1-2*A2*To \\ W2 &= 1-W4*To & W4 &= A7+A6 \end{aligned} \tag{2.2.13}$$

and we obtain the temperature function f3(T)

$$f3(T) = \frac{W1+W3*T+A2*T^2}{W2+W4*T} \tag{2.2.14}$$

f3(T) is of no dimension.
We insert the function f3(T) in Eq. 2.2.9 and obtain an expression for the temperature deviation dT.

2.2.15 $$dT = \frac{Ao*xo}{qo*A5*m} * i^2(t)*dt * f3(T)$$

We replace the mass m which does not depend on the temperature by A8*qo*xo and obtain a material constant C1(M) of the dimension $K*mm^4/(A^2*s)$.

$$C1(M) = Ao/(A5*A8)$$

Eq. 2.2.15 changes into the equation

2.2.16 $$dT = (C1(M)/qo^2) * i^2(t)*dt * f3(T)$$

2.3 Influence of the time on the temperature deviation

A time dependent current i(t) is considered as a product of a maximum value io and a time dependent function f(t). i(t) may be a single current pulse or a number of several different pulses. The shape of the pulses shall be known by calculation or by measurement. Then it is possible to have an approximation found by the function

2.3.1 $$i(t) = io*f(t)$$

We insert the current i(t) of Eq. 2.3.1 in Eq. 2.2.15 and obtain the temperature deviation dT

2.3.2 $$dT = (C1(M)/qo^2) * io^2 * f^2(t) * dt * f3(T)$$

The expression io/qo represents the maximum current density D in the wire at normal temperature and does not depend on the time and not on the temperature. The current density D depends on the cross section of the wire. Thus Eq. 2.3.2 yields the following equation for the temperature deviation dT.

2.3.3 $$dT = C1(M) * D^2 * f^2(t) * dt * f3(T)$$

$C1(M)*D^2 = C2(M)$ depends on material constants only and is of the dimension K/s. We rearrange and obtain the relation

2.3.4 $$dT/f3(T) = C2(M) * f^2(t) * dt$$

* The differential equation 2.3.4 is an equation in which a *
* pure temperature function f3(T) and a pure time function *
* f(t) are connected with an expression C2(M) of material *
* constants only.

2.4 Calculation of the temperature rise

Formula 2.3.2 enables us to determine the temperature rise of a single wire when a current pulse is going through.The length of the wire defines a maximum pulse length tm according to Eq. 1.9. The following deduction shall be applied only to such lengths of wires.
We rearrange the variables T and t of Eq. 2.3.2 on different sides of the equal sign.

$$dT/f3(T) = (Cl(M)/qo^2) * i^2(t) * dt \qquad (2.4.1)$$

and we integrate

$$\int_{To}^{Tn} \frac{dT}{f3(T)} = \frac{Cl(M)}{qo^2} * \int_{to}^{tn} i^2(t)*dt \qquad (2.4.2)$$

A current pulse flowing in a wire for the time tn causes the temperature T to rise from a value To up to a value Tn.
To shall be equal to or greater than 20 degrees centigrade and Tn shall be less than or equal to the melting temperature Ts.
The time function i(t) is known.The integral of the square of the current can be determined e.g. by the Simpson rule with sufficient accuracy.Cl(M) is known too and the product can be calculated.

For practical use the integral of the temperature is determined by means of a programmable calculator.

At first we specify the initial temperature value To at the moment to.Then we calculate the temperature value Tn by varying the value Tn until the value of the integral is very close to the value of the product of the right side of Eq. 2.4.2 .

An example will show some values: material : Ag
cross section : 1 mm^2
i^2*t-value : 20 000 A^2*s

table 3 :

Tn :	183	471	950	degrees centigrade
To :	20	200	500	degrees centigrade

2.5 Temperature rise up to the melting temperature

We will determine the amount of energy needed for the heating of a wire beginning at the temperature value To up to the value Ts of the melting temperature. No loss of heat shall be allowed. The melting temperature is known for every sort of metal and the value To is known by the evaluated experiment.
The function f3(T) is found according to chapter 2.2.

Eq. 2.4.2 is integrated from To to Ts. The value of this definite integral is a characteristic constant K1(M) for each conductor. The value K1(M) is of the dimension K.

$$2.5.1 \qquad K1(M) = \int_{To}^{Ts} (dT/f3(T))$$

For the deduction of f3(T) an acceptable simplification has been done due to Cu- and Ag-material. Hence the function f3(T) in Eq. 2.2.14 is valid only for Cu and Ag. When a different material is used the function f3(T) must be determined according to the deduction of chapter 2.2.
The value of the integral in Eq. 2.5.1 will be determined in a simple way by means of the Simpson rule. We find

$$2.5.2 \qquad K1(Ag) = 408 \text{ K} \qquad\qquad K1(Cu) = 441 \text{ K}$$

At the moment to a current i(t) begins to flow in the wire. At the moment tn the melting temperature Ts is reached. For this period we calculate the integral over the function f(t).

$$2.5.3 \qquad F1(t) = \int_{to}^{ts} f^2(t)*dt$$

F1(t) is of the dimension s.
We remember Eq.2.3.2 and Eq. 2.4.2. When the temperature reaches the melting point we obtain according to Eq. 2.4.2

$$2.5.5 \qquad K1(M)/C1(M) = K(M) = (io/qo)^2 * F1(t)$$

The deduction leads to a known result with the fact that K(M) is calculated. We have

2.5.6 $$K(M) = (io/qo)^2 * \int_{to}^{ts} f^2(t)*dt$$

K(M) is of the dimension A^2*s/mm^4

We apply the result to a wire of the cross section 1 mm^2 and we will have the important statement

* If we have a short current pulse i(t) and no heat losses *
* along the wire and the value of the integral of the *
* square of the current is equal to the value K(M),the wire *
* will be heated up to the melting point.

If there is no oxidation,this statement is equal to

* If a short current pulse i(t) results in a value less than *
* K(M) we will have no permanent deformation of the cross *
* section of the wire.This means no aging will occur. *

This statement is equal to the following too.

* If a short current pulse i(t) results in a value greater *
* than K(M) there may be permanent deformation of the cross *
* section of the heated wire.

Permanent deformation of the cross section of a heated wire means a change of the value of the cross section of the wire. When a strip is heated up by a current pulse then permanent deformation means either a change in the shape of the cross section with the value remaining constant or a change in both the shape and the value of the cross section.We may neglect the possibility of changing the value and not to change the shape of the cross section.

We can express the above mentioned facts in numerical values.

2.5.7 K(Ag) = 63 500 A^2*s/mm^4

K(Cu) = 82 100 A^2*s/mm^4

We note that only the i^2*t-value is important and not the shape of the current pulse.

2.6. The effect of the latent heat

The melting process consists on both the heating of the material up to the melting point and the transition from the solid into the liquid state. The amount of heat needed for the transition is a material constant $B1$.

During the transition solid/liquid the melting temperature Ts remains constant. The resitivity of the solid wire is $Rs = W9$. The resitivity of the liquid wire is $Rs = A9$. The transition from the value $W9$ to the value $A9$ is assumed to be steady and not by a jump. We will describe this by a simple approximation using a linear function $Rs = f(Q)$ of the applied heat Q. The heat Q is thought of as the heat per mass unit, $(W*s/g)$.

We will calculate with a model once more.

At the temperature To a wire shall have the length xo, the cross section qo and the mass m. The wire is heated up to the melting temperature Ts. The resistance rises up to the value $R(Ts)$. In the time dt a current pulse $i(t)$ is creating a heat dQ per mass unit in the wire at the temperature Ts.

2.6.1 $$dQ = (R(Ts)/m) * i^2(t) * dt$$

At the moment to no latent heat exists. At the moment $t1$ the wire has become liquid. The total heat created in the period $t2-t1$ is equal to the value $B1$. We may write the conditions

2.6.2 $t1 = 0 \qquad Q1 = 0 \qquad Rs = W9$

2.6.3 $t2 \qquad Q2 = B1 \qquad Rs = A9$

Hence we state the following simple relation between the heat Q needed for the transition solid/liquid and the resitivity Rs.

2.6.4 $$\begin{aligned} Rs &= W9 + (A9-W9)*Q/Q2 \\ &= W9 * \left(1+\frac{A9-W9}{W9}*\frac{Q}{Q2}\right) \end{aligned}$$

We obtain the resistance of the wire in the period $t2-t1$.

2.6.5 $$R(Ts) = Rs * \frac{x(Q)}{q(Q)} = W9 * \left(1+\frac{A9-W9}{W9}*\frac{Q}{Q2}\right) * \frac{x(Q)}{q(Q)}$$

We will have a linear function between the expression $x(Q)/q(Q)$ and the applied heat Q too.

At the melting point the solid wire has the density B3 and the liquid wire has the density B4. A simple linear approximation for the change in the density during the transition process is found by the formula

$$D(Q) = B3 + (B4-B3)*Q/Q2 = B3*(1 + (B4/B3-1)*Q/Q2) \tag{2.6.6}$$

We shorten (1 -B4/B3) = W5 and obtain the density D depending on the applied heat Q

$$D(Q) = B3 * (1 - W5*Q/Q2) \tag{2.6.7}$$

The mass m does not depend on the heat Q.Hence the change in the density is replaced by the change in the volume.
Only the first order approximation will be done.
For silver and copper material the value W5 is less than 0.1 .
We may write

$$1/(1-W5*Q/Q2) = 1 + W5*Q/Q2 \tag{2.6.8}$$

Therefore we obtain the volume depending on the applied heat Q.

$$V(Q) = V(Ts) * (1 + W5*Q/Q2) \tag{2.6.9}$$

A deviation W6 of the length equals approximately the third part of the deviation W5 of the volume, W6=W5/3 .
According to chapter 2.2 we obtain

$$\frac{x(Q)}{q(Q)} = \frac{x(Ts)}{q(Ts)*(1+W6*Q/Q2)} = \frac{xo}{qo*(1+A7*(Ts-To))*(1+W6*Q/Q2)} \tag{2.6.10}$$

Eq. 2.6.10 inserted in Eq. 2.6.5 yields the resistance of the wire at the temperature Ts during the transition solid/liquid.

$$R(Ts) = W9 * \left(1 + \frac{A9-W9}{W9}*\frac{Q}{Q2}\right) * \frac{x(Ts)}{q(Ts)*(1+W6*Q/Q2)} \tag{2.6.11}$$

We collect the expressions containing Q in Eq. 2.6.11 and establish a function f1(Q).

$$f1(Q) = \frac{B1 + \frac{A9-W9}{W9}*Q}{B1 + W6*Q} \tag{2.6.12}$$

The last equations inserted in Eq. 2.6.1 yield the part dQ of the latent heat created by the current i(t) in the wire at the moment t.

2.6.13 $$dQ = f1(Q) * \frac{W9}{m} * \frac{xo}{qo} * \frac{1}{1+A7*(Ts-To)} * i^2(t)*dt$$

We separate the variables t and Q and integrate. Then we obtain

2.6.14 $$\int_0^{B1} \frac{dQ}{f1(Q)} = \frac{W9}{A8*qo^2} * \frac{1}{1+A7*(Ts-To)} * \int_{t1}^{t2} i^2(t)*dt$$

The integral over the heat dQ during the transition solid/liquid is a material constant K(Q) of the dimension W*s/g .

2.6.15 $$K(Q) = \int_0^{B1} \frac{dQ}{f1(Q)}$$

The value of the integral of Eq. 2.6.15 is calculated by means of the Simpson rule. We find

K(Q)	Ag: 0.770 W*s/g	Cu: 1.290 W*s/g

We rearrange Eq. 2.6.14 and obtain according to Eq. 2.5.6

2.6.16 $$(io/qo)^2 * \int_{t1}^{t2} f^2(t)*dt = \frac{A8*K(Q)}{W9}*(1+A7*(Ts-To)) = Q(M)$$

Q(M) is of the dimension A^2*s/mm^4.

Eq. 2.6.16 gives an important result.

* A wire which is heated up to the melting temperature will *
* change from the solid into the liquid state if a short *
* current pulse i(t) with no heat losses along the wire has *
* a time integral of the square of the current equal to the *
* value Q(M).

We consider a wire with the cross section 1 mm^2, insert numerical values in Eq. 2.6.16 and obtain

2.6.17 $$Q(Ag) = 9840\ A^2*s/mm^4$$

$$Q(Cu) = 12800\ A^2*s/mm^4$$

2.7 The melting integral value for adiabatic process

We add both the heating up and the transition solid/liquid. This means to add Eq. 2.5.6 and Eq. 2.6.16 .We set ts=tl and we obtain

$$\left(\frac{io}{qo}\right)^2 * \int_{to}^{tl} f^2(t)*dt + \left(\frac{io}{qo}\right)^2 * \int_{tl}^{t2} f^2(t)*dt =$$

2.7.1 $$= \left(\frac{io}{qo}\right)^2 * \int_{t0}^{t2} f^2(t)*dt = K(M) + Q(M) = Kq(M)$$

We find the numerical values:

2.7.2 $Kq(Ag) = 73340\ A^2*s/mm^4$ $Kq(Cu) = 97900\ A^2*s/mm^4$

Then the dynamic forces of the current flowing in the wire cut off the liquid cross section.For the melting integral value the liquid material is not to be vapourised by additional heat.

When the value of the cross section is different from the value $1\ mm^2$ then Eq. 2.7.1 changes into

2.7.3 $$\int_{t0}^{t2} i^2(t)*dt = qo^2 * Kq(M)$$

The resulting values Kq(M) for Ag and Cu material have been found by many experiments to be correct.

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